

7 SET OPERATIONS

In Chapter One we looked at sets. In Chapter Six we looked at subsets and set types. Now we look at the set operations union, intersection and difference and see how sets may be combined to create new sets. We look at set operator precedence.

7.1 SET UNION

The union of two sets is itself a set with all the elements from both sets combined.

$$\{ 1, 2, 3 \} \cup \{ 2, 3, 4 \} = \{ 1, 2, 3, 4 \}$$

The union operator, \cup , looks a bit like the u in union - only there is no tail.

EXERCISE 7.1

Evaluate each of the expressions shown below.

1 $\{ 1, 2, 3, 5, 8, 13 \} \cup \{ 3, 5, 7, 11, 13 \}$

2 $\{ \textit{bigJ}, \textit{littleJ}, \textit{may}, \textit{pat}, \textit{alice} \} \cup \{ \textit{tom}, \textit{denise}, \textit{may} \}$

3 $\{ \textit{daisy}, \textit{buttercup} \} \cup \{ \textit{rani} \}$

4 $\{ \textit{daisy}, \textit{buttercup} \} \cup \textit{rani}$

5 $\{ \textit{daisy}, \textit{buttercup} \} \cup \{ \textit{buttercup}, \textit{daisy} \}$

7.2 SET INTERSECTION

The intersection of two sets is itself a set with elements that are common to both sets.

$$\{ 1, 2, 3 \} \cap \{ 2, 3, 4 \} = \{ 2, 3 \}$$

The intersection symbol, \cap , looks a bit like the n in intersection, only there is no tail.

EXERCISE 7.2

Evaluate each of the expressions shown below.

1 $\{ 1, 2, 3, 5, 8, 13 \} \cap \{ 3, 5, 7, 11, 13 \}$

2 $\{ \textit{bigJ}, \textit{littleJ}, \textit{may}, \textit{pat}, \textit{alice} \} \cap \{ \textit{tom}, \textit{denise}, \textit{may} \}$

3 $\{ \textit{daisy}, \textit{buttercup} \} \cap \{ \textit{rani} \}$

4 $\{ \textit{daisy}, \textit{buttercup} \} \cap \textit{rani}$

5 $\{ \textit{daisy}, \textit{buttercup} \} \cap \{ \textit{buttercup}, \textit{daisy} \}$

7.3 SET DIFFERENCE

The difference of two sets is itself a set with the elements of the second set removed from the first set.

$$\{ 1, 2, 3, 4 \} \setminus \{ 2, 3 \} = \{ 1, 4 \}$$

A useful trick here is to write out the first set, then cross out elements that are in the second set. For example:

$$\begin{aligned} \{ a, b, c, d \} \setminus \{ c, d, e, f \} &= \{ a, b, \cancel{c}, \cancel{d} \} \\ &= \{ a, b \} \end{aligned}$$

EXERCISE 7.3

Evaluate each of the expressions shown below.

1 $\{ 1, 2, 3, 5, 8, 13 \} \setminus \{ 3, 5, 7, 11, 13 \}$

2 $\{ \textit{bigJ}, \textit{littleJ}, \textit{may}, \textit{pat}, \textit{alice} \} \setminus \{ \textit{tom}, \textit{denise}, \textit{may} \}$

3 $\{ \textit{daisy}, \textit{buttercup} \} \setminus \{ \textit{rani} \}$

4 $\{ \textit{daisy}, \textit{buttercup} \} \setminus \textit{rani}$

5 $\{ \textit{daisy}, \textit{buttercup} \} \setminus \{ \textit{buttercup}, \textit{daisy} \}$

7.4 PRECEDENCE

Precedence is the priority given to operations; those with the highest priority are evaluated before those with the lower priority.

brackets () have the highest priority

then intersection \cap

then union \cup and difference \setminus , which have equal priority.

If in doubt, use brackets to express your intention.

EXERCISE 7.4

Evaluate each of the set expressions shown below.

1 $\{ 1, 2, 3 \} \cup \{ 2, 3, 4 \} \cap \{ 3, 4, 5 \}$

2 $(\{ 1, 2, 3 \} \cup \{ 2, 3, 4 \}) \cap \{ 3, 4, 5 \}$

3 $\{ 1, 2, 3 \} \cup \{ 2, 3, 4 \} \setminus \{ 3, 4, 5 \}$

4 $\{ 1, 2, 3 \} \setminus \{ 2, 3, 4 \} \cup \{ 3, 4, 5 \}$

5 $\{ 2, 3, 4 \} \setminus \{ 3, 4, 5 \} \cap \{ 1, 2, 3 \}$

REVIEW

We looked at the set operations union, intersection and difference.

The union of $\{ 1, 2, 3 \}$ and $\{ 2, 3, 4 \}$ is $\{ 1, 2, 3, 4 \}$ - the combined elements from both sets.

The intersection of $\{ 1, 2, 3 \}$ and $\{ 2, 3, 4 \}$ is $\{ 2, 3 \}$ - the elements in common to both sets.

The difference $\{ 1, 2, 3, 4 \} \setminus \{ 2, 3 \}$ is $\{ 1, 4 \}$ - the elements from the first set with elements in the second set removed .

We looked at the order of precedence: brackets first, then intersection, then union and difference in the order they appear in the expression.

Next we look at the Schema Calculus - how to create larger schemas from smaller ones by using conjunction and disjunction.

BIBLIOGRAPHY

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