6 SET TYPES

In Chapter One we looked at sets. In Chapter Two we looked at basic types and saw that a type is itself a set. A variable of a basic type represents just a single element drawn from the type set. Now we look at set types. A variable of a set type is itself a set and is used to represent collections such as a class of students, a herd of cows and a set of hotel rooms.

6.1 PROPER SUBSETS

In Chapter Three we introduced $\mathbb{Z}$, the set of all integers, negative, zero and positive. In Chapter Five we defined $\mathbb{N}$, the set of all positive integers including zero and excluding the negative integers.

We say that $\mathbb{N}$ is a proper subset of $\mathbb{Z}$ because every element in $\mathbb{N}$ is also in $\mathbb{Z}$ *and* $\mathbb{N} \neq \mathbb{Z}$. We write

$$\mathbb{N} \subset \mathbb{Z}$$

where $\subset$ means is-a-proper-subset-of.

A subset is part of a set, a set within a set.

EXERCISE 6.1

Which of the following expressions are true, which are false? Give reasons for your answers.

1 { *mon, wed, fri* } $\subset$ { *tue, thu, sat, mon, wed, fri, sun* }

2 { *Key* } $\subset$ { *hotKey, deleteKey, escapeKey* }

3 { 1, 2, 5, 7, 11 } $\subset$ { 11, 7, 5, 2, 1 }

4 { *memoryStick, compactDisk* } $\subset$ { *compactDisk, floppyDisk, memoryStick* }

5 { *tom, dee, harry* } $\subset$ { *anne, may, harry, dee, tom* }

6.2 SUBSETS

A subset is part of a set, a set within a set. *A* is a subset of *B* if every element in *A* is also in *B* *and* it is possible that *A* = *B*. We write

$$A \subseteq B$$

where $\subseteq$ means is-a-subset-of. To illustrate the point we list all the subsets of { 1, 2, 3 }. They are:

{ 1 } [the singletons]
{ 2 }
{ 3 }

{ 1, 2 } [the sets of pairs]
{ 1, 3 }
{ 2, 3 }

{ 1, 2, 3 } [because every element in { 1, 2, 3 }is also in { 1, 2, 3 }]

{ } [because if it was not, there would be an element in { } that is not in { 1, 2, 3 }. But { } has no elements. So { } must be a subset of { 1, 2, 3 }]

The empty set is a subset of every set.

If we make the following variable declaration:

$$x : \mathbb{F}\{ 1, 2, 3 \}$$

then *x* could be any one of the elements in

{ {}, { 1 }, { 2 }, { 3 }, { 1 ,2 }, { 1, 3 }, { 2, 3 }, { 1 ,2, 3 } }

where each element is itself a set. *x* could be { } or { 1 } or { 2 } or { 3 } or { 1, 2 } or { 1, 3 } or { 2, 3 } or { 1, 2, 3 }.

We say that *x* is a finite set of elements drawn from { 1, 2, 3 }. In short, *x* is a set.

Compare this with the declaration $y : \{ 1, 2, 3 \}$ which says *y* could be 1, or it could be 2, or it could be 3, and nothing else. In short, *y* is a single value from the set { 1, 2, 3 }. The difference in the two declarations is the presence or absence of fat $\mathbb{F}$.

EXERCISE 6.2

1 List the possible values of b if $b : \mathbb{F}\{ 0, 1 \}$

2 Explain the difference between a subset and a proper subset.

3 Explain the meaning of each of the expressions shown below:

a $a \in \{ a, b, c \}$

b $\{ a \} \subset \{ a, b, c \}$

c $\{ a \} \in \mathbb{F}\{ a, b, c \}$

6.3 SET TYPE DECLARATIONS

Access students study a number of subjects. Types here could include

STUDENT - the set of all students
SUBJECT - the set of all subjects

We might define

$accessStudents : \mathbb{F}\ STUDENT$
$accessCourse : \mathbb{F}\ SUBJECT$

and say *accessStudents* is a finite set of *STUDENT*s, *accessCourse* is a finite set of *SUBJECT*s.

An example of *accessStudents* is { *tom, deepak, harriet, anita, mayuri* }.

An example of an *accessCourse* is {*maths, formalMethods, programming, systemsAnalysis*}.

EXERCISE 6.3

1 Explain the difference between $\in$ and $\subseteq$.

2 Given [*PERSON*] the set of all people, explain the difference between

$p : PERSON$ and $q : \mathbb{F}\ PERSON$

3 A hotel system reserves rooms for guests from an arrival date to a departure date. Types here could include:

HOTEL - the set of all hotels
RESERVATION - the set of all reservations
ROOM - the set of all rooms
GUEST - the set of all guests
DATE - the set of all dates

Declare and explain variables to represent:

a a single hotel
b a finite set of rooms
c a single reservation
d a finite set of guests
e a single date.

REVIEW

We listed the subsets of a small set. We saw that set type variables store collections of similar objects.

Next we look at the set operations union, intersection and difference.

BIBLIOGRAPHY

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