3 THE INTEGER TYPE

In the last chapter we looked at types. A type is an inclusive set and all the elements of a set belong to the same type. We looked at declarations. A declaration introduces a variable and associates it with a type. We looked at how a consideration of types helps us to detect inconsistencies and errors.

Now we go on to look at the integer type in more detail. We review the arithmetic operations +, - and *, *div* and *mod*. We introduce *max* and *min*. We look at the numerical comparison operators <, ⩽, > and ⩾. And we look at rules of precedence.

3.1 THE ARITHMETIC OPERATORS

The type ℤ is the set of all possible integers.

... -3, -2, -1,0, 1, 2, 3, ...

We can add two integers together.	7 + 3 = 10	
We can subtract one from another.	10 - 7 = 3 7 - 10 = -3	
We can multiply two integers.	7 * 3 = 21	[* means multiply]
But we have a problem with division.	7 / 3 = 2.3333	[$2^1/_3$ = 2.3333]

2.3333 is not an integer.

Z provides the *div* and *mod* operators so the problem does not arise.

3.2 THE div AND mod OPERATORS

We may remember from our days at junior school, before we learned about decimal points and how to use a calculator, that:

$7 \div 3 = 2$ *remainder* 1 [because $7 = 3 \times 2 + 1$]

div (for integer division) works in a similar way to ÷ but without the remainder. So

7 div 3 = 2

div gives the integer result after dividing one integer by another. Any remainder or fractional part is *truncated*, cut off, lost.

mod (for modulo) works in a way similar to remainder. So

7 mod 3 = 1

mod gives the integer remainder after dividing one integer by another.

We remember that division by zero *is not defined*. So both

7 div 0 and 7 mod 0

have no defined answer; they are indeterminate.

div and mod are part of the Z Notation.

EXERCISE 3.1

1 Evaluate

a 16 div 3 **b** 20 div 4 **c** 5 div 5 **d** 4 div 5 **e** 7 div 5

2 Evaluate

a 16 mod 3 **b** 20 mod 4 **c** 5 mod 5 **d** 4 mod 5 **e** 7 mod 5

3.3 ARITHMETIC OPERATOR PRECEDENCE

Precedence is the order in which operations are always carried out. In arithmetic:

brackets have the highest priority

then *, div and mod all have equal priority

then + and - all have equal priority

For example:

(40 - 32) * 5 div 9	=	8 * 5 div 9	[brackets first]
	=	40 div 9	[multiply]
	=	4	[integer division]

EXERCISE 3.2

Evaluate

1 5 + 7 * 9

2 (5 + 7) * 9

3 5 div 9 * (212 - 32)

4 Given the declaration $n : \mathbb{N}$ and the observation n div 2 = 0, what can you conclude about n? You could copy and complete the table shown below.

n	n div 2	n div 2 = 0
0		
1		
2		
3		
4		
5	2	false

5 Given the declaration $n : \mathbb{N}$, what is the set of possible results of n mod 5? You could copy and complete the table shown below.

n	n mod 5
0	
1	
2	
3	
4	
5	

3.4 max AND min

Z also provides the *max* and *min* operators. *max* gives the largest value in a *non-empty set* of integers, and *min* the least value. For example:

max{ 11, 7, 13, 3, 5 } = 13

min{11, 7, 13, 3, 5 } = 3

min{ } *is undefined*

EXERCISE 3.3

Evaluate

1 *max*{ 3, 4, 7, 1, 2 }

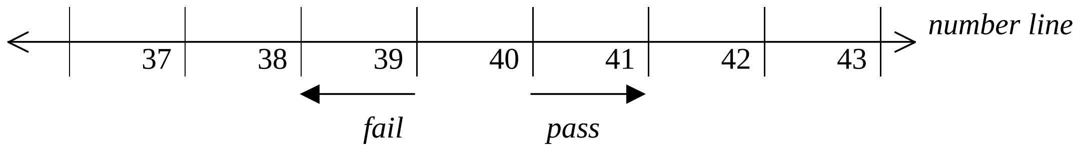
2 *min*{ -1, 0, 1 }

3 *max*{ -1, 0, 1 }

4 *max*{ }

3.5 THE NUMERICAL COMPARISON OPERATORS

40% is a critical mark in some exams. A mark of 40 or more is a pass. A mark of 39 or less is a fail. The boundary between fail and pass is shown below on the number line.



39 is-less-than 40. We write $39 < 40$. $<$ means is-less-than.

41 is-more-than 40. We write $41 > 40$. $>$ means is-more-than.

$<$ and $>$ are known as *comparison* or *relational* operators. They are easy to remember. **L**ess than has its point on the **L**eft. Mo**R**e than has its point on the **R**ight.

Evidently

$39 < 40$	is true
$40 < 39$	is false

And

$40 > 39$	is true
$39 > 40$	is false

$\leqslant$ means less-than-or-equal-to. $\geqslant$ means more-than-or-equal-to. So

$39 \leqslant 40$	is true
$40 \leqslant 40$	is true
$41 \leqslant 40$	is false

And

$40 \geqslant 39$	is true
$40 \geqslant 40$	is true
$40 \geqslant 41$	is false

EXERCISE 3.4

1 Given the declaration $n : \mathbb{Z}$ and that $n \geqslant 10$ state whether each of the following expressions are true or false:

a $n = 9$
b $n = 10$
c $n = 11$

2 Given the declaration $n : \mathbb{Z}$ and that $n < 100$ state whether each of the following expressions are true or false:

a $n = 90$
b $n = 100$
c $n = 101$

3 Given the declaration $n : \mathbb{Z}$ #and that $n \geqslant 0$ and $n < 5$ are both true, write down the set of values that n belongs to.

4 Given the declaration $n : \mathbb{N}$ and that $n \text{ div } 10 > 0$ and $n \text{ div } 10 \leqslant 1$ are both true, write down the set of values that n belongs to.

5 Given the declaration $n : \mathbb{Z}$ and that $n < 0$ and $n > 9$ are both true, what can you conclude about the values of n?

REVIEW

We looked at the integer type in more detail. We reviewed the arithmetic operations +, - and *, div (integer division) and mod (integer remainder) We introduced *max* and *min* - the largest and least values in a non-empty set of integers. We looked at the numerical comparison operators < (less than), ⩽ (less than or equal to), > (more than) and ⩾ (more than or equal to). And we looked at rules of precedence (brackets, multiplication and division, addition and subtraction) for the arithmetic operations.

Next, we see how the values of variables may be constrained by predicates.

BIBLIOGRAPHY

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