1 SETS

Sets are the foundation of all mathematics and our attempts to describe the world about us. We look at the properties of sets and some simple ways of describing them. We explain that a set is a collection of similar objects. We see how to display a set. We see that a set has no duplicates and no order. We examine a convention for naming sets. We look at some standard sets. We learn how to define number ranges. We examine the concepts of cardinality, membership and equality.

1.1 OBJECTS AND SETS

An object is anything we can see, smell, hear, taste, touch or think about. A collection of similar objects is called a *set*. Examples of sets include:

- the collection of people who have enrolled on this year's Formal Methods course
- the collection of leap years since 1986
- the collection of three letter passwords that can be generated from the letters a, b and c

EXERCISE 1.1

Identify three sets drawn from your surroundings and experience.

1.2 SET DISPLAY

One way to describe a set is to list its contents. For example:

{ *tom, anne, jerry* }

{ *1988, 1992, 1996, 2000, 2004* }

{ *abc, bca, cab, acb, bac, cba* }

We use curly brackets to separate the list from its surroundings. The objects that make up a set are known as its *elements* or *members*. We separate one member from the next by a comma.

Describing a set by listing its contents is called *set display*.

A set is defined by its contents alone. So the order in which you write the elements in a set display is not important. It makes no sense to talk about the first or last element in a set, or to say that one element comes before another.

{ *tom, anne, jerry* } is the same as { *jerry, anne, tom* }

If two elements in a set are the same, the repeated element is ignored. Writing the same element more than once is harmless but confusing. So we leave out repeats.

{ *tom, tom, jerry, anne* } is better written { *tom, jerry, anne* }

EXERCISE 1.2

Use set display notation to describe the sets given below.

1 the set of days you are timetabled to be in college.

2 the set of digits used in base 10 arithmetic (denary).

3 the set of digits used in base two arithmetic (binary).

4 the set of colours seen on a rainbow.

5 the set of departments in a small, manufacturing business.

1.3 NAMING A SET

If a set has too many members to conveniently list, we can simply give them descriptive names.

PERSON - the set of all people

LEAPYEAR - the set of all leap years ever

PASSWORD - the set of all possible passwords

We follow the convention that names we choose for our sets:

- are written entirely in capital letters - *PERSON* not Person
- are singular - *PASSWORD* not PASSWORDS
- do not contain spaces, underscores or hyphens - *LEAPYEAR* not LEAP YEAR

EXERCISE 1.3

Evaluate the names for the set of book titles shown below against the convention for naming sets described in section 1.3 above.

1 *BOOKTITLE*

2 *BOOK TITLE*

3 *BookTitle*

4 *BOOK_TITLE*

5 *BOOKTITLES*

1.4 SOME STANDARD SETS

Some sets have names already assigned to them.

ℤ (say fat zed) - the set of all whole numbers, negative, zero and positive. -1, 0, 1 are members of this set.

ℕ (say fat en) - the set of natural whole counting numbers from zero upwards. 0, 1, 2 are members of this set.

∅ - the empty set, the set with no members. Think of an empty bag - there is nothing in it.

Z and ℤ are not the same thing. Z is the name of a notation that uses maths to specify computer systems. ℤ is the set of all whole numbers. Whole numbers are also known as *integers*.

EXERCISE 1.4

Explain the difference between:

1 Z and ℤ

2 ∅ and { }

1.5 NUMBER RANGE

If a sequence of integers forms a set, we can use *number range notation*.

> 1..7 (say 1 up to 7) is the set of all integers between 1 and 7 inclusive.

1..7 is the set *{* 1, 2, 3, 4, 5, 6, 7 *}*. Note that there are just *two dots* between the two integers that mark the beginning and end of the sequence.

3..1 is an empty set because the first integer must be less than (or equal to) the second integer; you cannot count from 3 up to 1.

EXERCISE 1.5

Use number-range notation to define the sets described below.

1 the number of people who may be allowed in a lift if its capacity is five people

2 the set of hours used in 24-hour clock notation

3 the set of numbers that define any person's age in years

4 the set containing just the element 5

5 the empty set

1.6 CARDINALITY

Some sets contain more members than we can count. Think of ℤ, the set of all possible integers for example. No matter how many we count, there is always another one to be counted. Such sets are called *infinite sets.*

Some sets contain a finite number of elements - we can count the number of elements they contain. Look at the set { *tom, anne, jerry* }. We can see it has three elements.

The number of elements in a finite set is known as its *cardinality*. The symbol for cardinality is #. So for example:

#{ *tom, anne, jerry* } = 3

#{ *1988, 1992, 1996, 2000, 2004* } = 5

#{ *abc, bca, cab, acb, bac, cba* } = 6

#ℤ is undefined

EXERCISE 1.6

State the value of:

1 #{ *spaceKey, tabKey, returnKey, arrowKey, functionKey* }

2 #{ 314, 497, 573, 272 }

3 #0..5

4 #∅

5 #ℕ

1.7 MEMBERSHIP

If we look at the set { 1988, 1992, 1996, 2000, 2004 } we can see that 2000 is an element of the set but 2001 is not. We write

$$2000 \in \{ 1988, 1992, 1996, 2000, 2004 \}$$

where $\in$ means is-a-member-of

And

$$2001 \notin \{ 1988, 1992, 1996, 2000, 2004 \}$$

where $\notin$ means is-not-a-member-of.

EXERCISE 1.7

Which of the following expressions are true and which are false? Explain why.

1 *end* $\in$ { *if, else, while, repeat, until, end* }

2 *Key* $\in$ { *spaceKey, tabKey, returnKey, arrowKey, functionKey* }

3 $0 \notin \varnothing$

4 $3.142 \in \mathbb{Z}$

5 $-1 \in \mathbb{N}$

1.8 EQUALITY

Two sets are *equal* if they both have exactly the same elements. (We remember that the elements can be in any order and that repeated elements are ignored.) For example

$$\{ 1988, 1992, 1996, 2000, 2004 \} = \{ 2000, 1996, 1988, 2004, 1992 \}$$

= means is-the-same-as.

But if two sets do not have exactly the same elements, they are not equal. For example

{ 1988, 1992, 1996, 2000, 2004 } and { 1988, 1992, 1996, 2000 }

are not equal because 2004 is a member of one set but not the other. We write:

$$\{ 1988, 1992, 1996, 2000, 2004 \} \neq \{ 1988, 1992, 1996, 2000 \}$$

≠ means is-not-the-same-as.

EXERCISE 1.8

Which of the following expressions are true and which are false? Explain why.

1 { *robin, thrush, starling, sparrow, blackbird* } = { *wren, swallow, hawk, crow, magpie* }

2 $\{ 0, 1 \} \neq \{ 1, 0 \}$

3 $\{ 1, 2, 4, 8, 16 \} = \{ 16, 8, 4, 2, 1 \}$

4 { *carrots, onions, garlic* } ≠ { *garlic, onions, carrots, garlic* }

5 { *C, Smalltalk, Java, Eiffel, C++* } = { *C++, Eiffel, Java, Smalltalk, C* }

REVIEW

We defined what a set is - a collection of similar objects. We saw how to display a set - comma-separated elements within curly braces. We learned that a set has no duplicates and no order. We saw how to choose names for a set - singular, capitals, no spaces. We looked at some standard sets - $\mathbb{Z}$, $\mathbb{N}$ and $\varnothing$. We learned how to define number ranges. We examined the concepts of cardinality, membership and equality.

Next, we look at basic types.

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