

# The Z Notation

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## *Introduction*

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## ***Introduction***

Professionals have been writing computer programs for over 50 years and still they write programs that contain errors. The errors may range from annoying to serious. You may have experienced a word processor freezing causing you to lose some typing. Or you may have read of an aircraft crash caused by software failure and resulting in loss of life.

Professional programmers turn to mathematics and simple formal logic to discover errors and to prevent them occurring in computer programs. A *formal method* uses mathematics to specify, design, write and prove the correctness of computer programs. The Z Notation is a formal method. It was developed by the Programming Research Group at Oxford University (and others) in the 1970's and has been used world-wide ever since.

These Z Notes will introduce you to the logic and mathematics used by programmers for specifying, designing and writing robust computer programs.